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The triangle can work wonders as three is not an even number

Here we look at further positions with king and two pawns vs king and one pawn. If the extra pawn is a passed pawn but not a protected passed pawn, the attacker also usually wins easily. The passed pawn deflects the defending king and the attacking king can win the defender’s pawn and then win with the remaining pawn. But there are difficult cases, especially when blocked rook’s pawns are involved. With an additional bishop’s pawn, the win using triangulation is worth knowing. For the case that the passed pawn is further away, see Lesson 15.

The final four diagrams show how the kings fight for vital squares when there are no passed pawns.

1) White to move
White wins by transferring the move to Black: 1 Êe5 Êf8 (Black can’t mirror the manoeuvre because e7 is covered by the f6-pawn) 2 Êf4 Êe8 3 Êe4 (2). This triangle manoeuvre puts Black in a fatal zugzwang.

2) Black to move
White wins since ...Êg8 is illegal, and 3...Êf7 loses directly to 4 Êf5 Êf8 5 Êg6. That leaves 3...Êf8 4 Êe5 Êe8 5 Êe6 Êf8 6 Êf7 Êg7 7 Êe7 Êh7 8 Êf6, when White wins.
3) White to move

The triangle also helps here: 1 Êb3 Êc6 2 Êb2 Êd6 3 Êc2 (the triangle is completed) 3...Êd5 4 Êc3 Êc6 5 Êd4 Êd6 6 a3 (a vital ‘tempo move’ to put Black in zugzwang) 6...Êc6 7 Êe5 and White wins.

4) Black to move

In general the attacker can’t use triangulation in such situations (Black can also ‘triangulate’ in reply) and opposition is the main fighting method for the key squares: 1...Êg7! (5) (for 1...Êg5?, see diagram 6).

5) White to move

Black has taken the distant opposition, and draws: 2 Êf3 Êf7! 3 Êf4 Êf6 4 Êe3 Êe7 5 Êd4 Êd6 Êc3 Êc7 7 Êb4 Êb6.

6) White to move

White wins with 2 Êf3 Êf5 3 Êe3 Êe5 4 Êd3 Êf6 (Black can’t get to the queenside in time) 5 Êc3 Êe7 6 Êb4 Êd7 7 Êb5 Êc7 8 Êa6.
Now we consider a pair of blocked pawns with an extra pawn for the attacker several files away. If the attacking king can support the passed pawn, he usually wins (see also Lesson 14). But there is an important exception, if the blocked pawns are on a rook’s file. Let’s assume they are on the a-file. If the extra pawn is on the d-, e-, f-, g- or h-file then Bähr’s Rule can help us assess the position.

This rule has two parts. Both kings must be in position near the pawn as in the first diagram below. If the attacker’s rook’s pawn has crossed the middle of the board, the attacker wins. If not then draw a diagonal from the defender’s pawn towards the eighth rank and from the point of intersection with the c-file back towards the first rank. (This sounds complex but the diagrams should make it clearer.) If the attacking passed pawn has not crossed this diagonal, then the attacker wins.

1) White to move
For the correct 1 d4!, see diagram 3. Instead, 1 f5+? is a typical mistake: 1...f6 2 f4 f7 3 e5 e7 (2).

2) White to move
This is drawn, since the f-pawn has crossed the c7-h2 diagonal: 4 d5 f6 5 c5 xf5 6 b5 e6 7 xa5 d7 8 b6 c8 with a draw (see Lesson 9).
3) Black to move
White wins the race of the kings, as Bähr’s Rule predicts: 1...\textit{f}5 2 \textit{c}5 \textit{xf}4 3 \textit{b}5 \textit{e}5 4 \textit{xa}5 \textit{d}6 5 \textit{b}6 \textit{d}7 6 \textit{b}7, winning.

4) White to move
The a5-pawn has crossed the middle of the board, so White wins as a rule. But the direct 1 \textit{d}4? runs into 1...\textit{xf}4 2 \textit{c}5 \textit{e}5 3 \textit{b}6 \textit{d}6 4 \textit{xa}6 \textit{c}7, drawing. The king must first gain space: 1 \textit{f}3! \textit{f}6 2 \textit{e}4 \textit{e}6 (5).

5) White to move
Now White can head for the a-pawn: 3 \textit{d}4 \textit{f}5 4 \textit{e}5 \textit{xf}4 5 \textit{b}6 \textit{e}5 6 \textit{xa}6 \textit{d}6 7 \textit{b}7, winning.

6) White to move
There are exceptions if the passed pawn is far behind. Here a counterattack saves Black: 1 \textit{e}3+ \textit{c}4 2 \textit{e}2 \textit{b}4 3 \textit{e}4 \textit{xa}5 4 \textit{e}5 \textit{b}6, with a draw.